Gilles
8/12/2011

Open questions:

Recall Ribe: $X = \left(\sum \ell_{1+\frac{1}{n}}\right)_{\ell_2}$: $\ell_1 \oplus X \underset{u.H}{\sim} X$

reflexive space : $\ell_1 \underset{u.c}{\hookrightarrow} X$

Assume $X \hookrightarrow Y$ reflexive. What other assumptions will ensure that X is reflexive?

<u>Th</u>: $\ell_1^n \subseteq X$ and $X \underset{c}{\hookrightarrow} Y$ reflexive $\Rightarrow$ X reflexive.

or X is an "alternate-Banach-Saks-property" space.

<u>Counterexample</u>: $J, J^* \underset{c}{\not\hookrightarrow} Y$ reflexive.

Open question: ① Is there a quasi-reflexive $(\dim X^{**}/X < \infty)$ non reflexive B-space X such that $X \underset{c}{\hookrightarrow} Y$ reflexive?

② $X \underset{c}{\hookrightarrow} Y$ reflexive $\not\Rightarrow$ X is weakly sequentially complete

③ X^* separable + $X \underset{c}{\hookrightarrow} Y$ reflexive $\not\Rightarrow$ X reflexive.

(you can try to add some Szlenk index property.)