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Introduction to noncommutative analysis

This could be the title of 5 unrelated talks

Singular trace

Alain Connes played a main rôle here (1994) (discussions w. Jacques Dixmier)
"quantized calculus" $\rightsquigarrow$ Connes' trace theorem.
(cf. book by Edwards on good versions of Hahn-Banach theorems.

1931 Sargent
There is a new proof:
Wasitzky

Introduction to nc analysis. 2^{nd} lecture. Sukochev

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Let $\mathcal{L}_{1,\infty} = \{ A \in B(H) : \mu(t,A) \leq \frac{C}{t} \}$

$\mathcal{M}_{1,\infty} = \{ \quad : \int_0^t \mu(s,A) \leq C \log(1+t) \}$.

If $B \in \mathcal{M}_{1,\infty}$ and $A \prec\prec B$, then $A \in \mathcal{M}_{1,\infty}$.

Submajorisation $\prec\prec$ has a strong link to interpolation theory:

$E \in Int_1(\mathcal{L}_1, \mathcal{L}_\infty) \Leftrightarrow \left(B \in E,\ A \prec\prec B \Rightarrow A \in E \text{ and } \|A\|_E \leq \|B\|_E \right)$

E symmetric means $\overset{A,B \in E}{\mu(A) \leq \mu(B)} \Rightarrow \|A\|_E \leq \|B\|_E$

E fully symmetric means $A \prec\prec B$, $B \in E \Rightarrow A \in E$ and $\|A\|_E \leq \|B\|_E$.

it just means $\|AXB\|_E \leq \|A\|_\infty \|X\|_E \|B\|_\infty$.

Cf Lorentz Acta math 1948 : action of translation invariant limits on l^∞

He introduced "almost convergence" by "$l(x)$ does not depend on the Banach limit l".

Cf Pietsch's results: Dixmier traces and Banach limits are in 1-1 correspondance for $\mathcal{L}_{1,\infty}$.

Q: what are the extreme points of the Banach limits?

Cf. submitted Article with Semenov.