Spatial Quantum Dynamics

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Markov semigroups, E_0–semigroups, product systems

Irreversible dynamics, reversible dynamics, and dilations

- Irreversible dynamics: Markov semigroups $T = (T_t)_{t \in \mathbb{R}_+}$

$$T_t \colon \mathcal{B} \xrightarrow[\text{CP}]{\text{unital}} \mathcal{B}$$

- Semireversible dynamics: E_0–Semigroups $\vartheta = (\vartheta_t)_{t \in \mathbb{R}_+}$

$$\vartheta_t \colon \mathcal{A} \xrightarrow[\text{endomorphisms}]{\text{unital (faithful)}} \mathcal{A}$$

- (Reversible dynamics: Automorphism groups $\alpha = (\alpha_t)_{t \in \mathbb{R}_+}$)

$\mathcal{A}, \mathcal{B}, \ldots$ unital C^*–algebras. If von Neumann, then all maps normal.

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$$T_t \colon \mathcal{B} \xrightarrow[\text{CP}]{\text{unital}} \mathcal{B}$$

- Semireversible dynamics: E_0–Semigroups $\vartheta = (\vartheta_t)_{t \in \mathbb{R}_+}$

$$\vartheta_t \colon \mathcal{A} \xrightarrow[\text{endomorphisms}]{\text{unital (faithful)}} \mathcal{A}$$

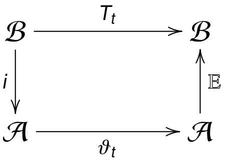
- Dilation:

$$\begin{array}{ccc} \mathcal{B} & \xrightarrow{\;T_t\;} & \mathcal{B} \\ {\scriptstyle \text{embedding } i}\big\downarrow & & \big\uparrow {\scriptstyle \mathbb{E} \text{ (expectation: } i \circ \mathbb{E} \text{ is cond. exp.)}} \\ \mathcal{A} & \xrightarrow[\;\vartheta_t\;]{} & \mathcal{A} \end{array}$$

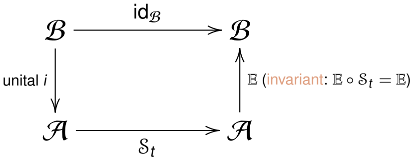
Markov semigroups, E_0–semigroups, product systems

Dilations, noises, and cocycle perturbations

Dilation:



Noise:
(Interaction picture)
(Conditional independence)



Unitary left cocycle: Unitaries $u_t \in \mathcal{A}$ such that $u_{s+t} = u_t \mathcal{S}_t(u_s)$

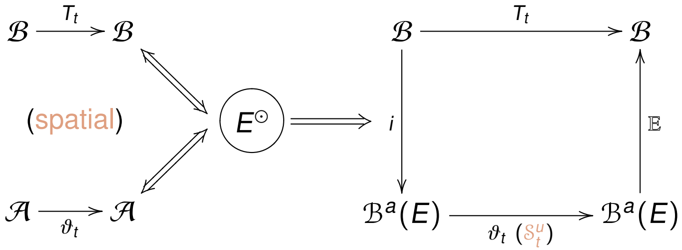
$\Longrightarrow \quad \mathcal{S}^u_t := u_t \mathcal{S}_t(\bullet) u_t^*$ is E_0–semigroup (cocycle perturbation)

A basic problem of quantum probability:

Given T, find $\mathcal{S}$ and u_t such that $\vartheta := \mathcal{S}^u$ is a dilation.

Markov semigroups, E_0–semigroups, product systems

Semigroups and product systems



- $T \Longrightarrow E^{\odot}$: Via Paschke's GNS-construction for T_t! (1973!)
- Special case: ϑ_t on $\mathcal{A}$ $\rightsquigarrow$ ${}_t\mathcal{A} = {}_{\vartheta_t}\mathcal{A}$. $(a.a' := \vartheta_t(a)a'.)$
- $\vartheta \Longleftrightarrow E^{\odot}$ more interesting for $\mathcal{A} = \mathcal{B}^a(E)$. (Arveson for H.)
- Finally, also T on $\mathcal{B}^a(E)$ plays a role.

Also CP-semigroups and E–semigroups.
Also multi-parameter case. (Or more general semigroups.)
($\Rightarrow$ relations with multivariate operator theory.)

Definition (Powers 1987 ($\mathcal{B}(H)$), MS 2006 (preprint 2001))

An E_0–semigroup ϑ is spatial if it admits a semigroup of intertwining isometries (u_t): $\quad \vartheta_t(a)u_t = u_t a$.

Definition (Arveson 1997 ($\mathcal{B}(H)$); Bhat-Liebscher-MS 2010)

A CP-semigroup T is spatial if admits a (sufficiently continuous!) semigroup (c_t) such that $T_t - c_t^* \bullet c_t$ is CP for all t.

(**Note:** Powers 2004 requires semigroup of intertwining isometries, which is more restrictive.)

Definition (MS 2006 (preprint 2001))

A spatial product system is a pair $(E^\odot, \omega^\odot)$ consisting of a PS $E^\odot$ and central unital reference unit $\omega^\odot$.
Unital: $\langle \omega_t, \omega_t \rangle = \mathbf{1}$. $\quad$ *Central*: $b\omega_t = \omega_t b$.

Outline

Product systems, units, and CP-semigroups

Product systems, E_0–semigroups, and noises

Unital units and weak dilations of Markov semigroups

Product systems and units

Recall: $E = {}_{\mathcal{A}}E_{\mathcal{B}}$ and $F = {}_{\mathcal{B}}F_{\mathcal{C}}$ correspondences
$\rightsquigarrow$ tensor product ${}_{\mathcal{A}}E \odot F_{\mathcal{C}}$ generated by $x \odot y$ subject to

$$\langle x \odot y, x' \odot y' \rangle \;=\; \langle y, \langle x, x' \rangle y' \rangle, \qquad a(x \odot y) \;=\; (ax) \odot y.$$

Definition

A product system is a family $E^\odot = (E_t)_{t \in \mathbb{R}_+}$ of correspondences over $\mathcal{B}$ with $E_0 = \mathcal{B}$ and bilinear unitaries $u_{s,t} \colon E_s \odot E_t \to E_{s+t}$, such that:

- *The "product" $x_s y_t := u_{s,t}(x_s \odot y_t)$ is associative.*
- *$u_{0,t}$ and $u_{t,0}$ reduce to the left and right action of $E_0 = \mathcal{B}$ on E_t.*

Definition

A unit $\xi^\odot = (\xi_t)_{t \in \mathbb{R}_+}$ is a section such that $\xi_s \xi_t = \xi_{s+t}$ and $\xi_0 = \mathbf{1}$.

Units and CP-semigroups

Consequence:

$$\langle \xi_{s+t}, b\xi_{s+t} \rangle = \langle \xi_s \odot \xi_t, b\xi_s \odot \xi_t \rangle = \langle \xi_t, \langle \xi_s, b\xi_s \rangle \xi_t \rangle,$$

$\rightsquigarrow$ the maps $T_t := \langle \xi_t, \bullet \xi_t \rangle$ form an (obviously CP-)semigroup.

Definition

A pair $(E^\odot, \xi^\odot)$ is the (unique) GNS-construction of a CP-semigroup T if $T_t = \langle \xi_t, \bullet \xi_t \rangle$ and if $\xi^\odot$ generates $E^\odot$.

Theorem (Bhat-MS 2000)

Every (one-parameter) CP-semigroup (on a necessarily unital C^*–algebra) has a GNS-construction.

Note: T unital $\iff$ $\xi^\odot$ unital ($\langle \xi_t, \xi_t \rangle = \mathbf{1}$).

Spatial CP-semigroups

Suppose GNS-T embeds into spatial PS: $(E^\odot, \xi^\odot, \omega^\odot)$.
Then $c_t := \langle \omega_t, \xi_t \rangle$ form semigroup in $\mathcal{B}$ and $T_t - c_t^* \bullet c_t$ is

$$\langle \xi_t, \bullet \xi_t \rangle - \langle \xi_t, \omega_t \rangle \bullet \langle \omega_t, \xi_t \rangle = \langle (\mathrm{id} - \omega_t \omega_t^*) \xi_t, \bullet (\mathrm{id} - \omega_t \omega_t^*) \xi_t \rangle,$$

and, clearly, CP, so T is spatial.
Note: Strongly continuous units in (strongly) continuous PS, then all semigroups

$$\begin{pmatrix} \mathfrak{T}_t^{0,0} & \mathfrak{T}_t^{0,1} \\ \mathfrak{T}_t^{1,0} & \mathfrak{T}_t^{1,1} \end{pmatrix} := \begin{pmatrix} \langle \omega_t, \bullet \omega_t \rangle & \langle \omega_t, \bullet \xi_t \rangle \\ \langle \xi_t, \bullet \omega_t \rangle & \langle \xi_t, \bullet \xi_t \rangle \end{pmatrix} = \begin{pmatrix} \mathrm{id}_{\mathcal{B}} & \bullet c_t \\ c_t^* \bullet & T_t \end{pmatrix}$$

are strongly continuous.

- C^***–case:** c_t is norm continuous and T has CE-generator. (Defi. cont. PS by MS 2003 (preprint 2001).)
- **vN-case:** Much more interesting. (Much weaker topology!) (Defi. strongly cont. PS by MS 2009 (preprint).)

Spatial CP-semigroups

Theorem (Bhat-Liebscher-MS 2010)

C^*–case: *A strongly continuous CP-semigroup is spatial iff its GNS-system embeds into a continuous spatial product system.*

Proof of "only if": Suppose strongly continuous T_t dominates $c_t^* \bullet c_t$ for continuous c_t. Then

$$\begin{pmatrix} \mathfrak{T}_t^{0,0} & \mathfrak{T}_t^{0,1} \\ \mathfrak{T}_t^{1,0} & \mathfrak{T}_t^{1,1} \end{pmatrix} := \begin{pmatrix} \mathrm{id}_{\mathcal{B}} & \bullet c_t \\ c_t^* \bullet & T_t \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & T_t - c_t^* \bullet c_t \end{pmatrix} + \begin{pmatrix} \mathrm{id}_{\mathcal{B}} & \bullet c_t \\ c_t^* \bullet & c_t^* \bullet c_t \end{pmatrix},$$

is a strongly continuous CP-semigroup on $M_2(\mathcal{B})$.
Barreto-Bhat-Liebscher-MS 2004 and MS 2009 (preprint):
There exists continuous $(E^\odot, \xi^\odot, \omega^\odot)$. □

Theorem (MS 2009 (preprint))

vN-case: *A strongly continuous CP-semigroup is spatial iff its GNS-system is a strongly continuous spatial product system.*

Spatial *versus* Fock

General:

- Spatial+uniformly continuous $\Rightarrow$ GNS $\subset$ Fock.
- (More precisely: A continuous set of units among which there is one $\omega^{\odot}$ generates Fock.)

C*–case:

- Spatial $\Rightarrow$ uniformly continuous $\Rightarrow$ GNS $\subset$ Fock.
- Barreto-Bhat-Liebscher-MS 2004: Uniformly continuous (=:type I) $\not\Rightarrow$ GNS $\subset$ Fock.
- Bhat-Liebscher-MS 2010: GNS $\subset$ Fock $\not\Rightarrow$ GNS $=$ Fock.

vN-case (Barreto-Bhat-Liebscher-MS 2004):

- Uniformly continuous $\Rightarrow$ spatial.
  (Equivalent to Christensen-Evans 1979. Cf. Raja's talk.)
- Sub Fock $=$ Fock.
- Uniformly continuous $\Rightarrow$ spatial $\Rightarrow$ GNS $=$ Fock.
- MS 2009 (preprint): Spatial $\Rightarrow$ GNS spatial.

E_0–Semigroups and product systems

- $\mathcal{B}$ a C^*–algebra. $E = E_{\mathcal{B}}$ a Hilbert $\mathcal{B}$–module.
- For convenience: E full, that is, $\overline{\operatorname{span}}\langle E, E\rangle = \mathcal{B}$.
- $\vartheta = (\vartheta_t)_{t\geq 0}$ strict E_0–semigroup on $\mathcal{B}^a(E)$.
  Strict: $\overline{\operatorname{span}}\,\vartheta_t(EE^*)E = E$.

Theorem (Bhat 1996 ($\mathcal{B}(H)$), MS 2002, 2009 (preprint 2004))

There is a unique full product system $E^{\odot}$ and a family of unitaries $v_t \colon E \odot E_t \to E$ such that $\vartheta_t(a) = v_t(a \odot \mathrm{id}_t)v_t^*$.

- With $xy_t := v_t(x \odot y_t)$ we have $(xy_s)z_t = x(y_s z_t)$, that is, the v_t are a left dilation of (full!) $E^{\odot}$ to (full!) E.
- Each left dilation v_t defines E_0–semigroup $\vartheta_t := v_t(\bullet \odot \mathrm{id}_t)v_t^*$.

Theorem (Arveson 1990, MS 2006 ($\mathcal{B}(H)$), MS variants)

Every full product system admits a left dilation.

Theorem

- *MS 2002: Two E_0–semigroups ϑ, ϑ' on $\mathcal{B}^a(E)$ are cocycle equivalent ⇔ they have isomorphic PS.*
- *MS 2009 (preprint): Two E_0–semigroups ϑ, ϑ' on $\mathcal{B}^a(E)$ are cocycle conjugate ⇔ they have Morita equivalent PS.*

Theorem (MS 2009 (preprint))

- *E_0–semigroups ϑ on $\mathcal{B}^a(E)$ and ϑ' on $\mathcal{B}^a(E')$ are stably cocycle equivalent ⇔ they have isomorphic PS.*
- *E_0–semigroups ϑ on $\mathcal{B}^a(E_\mathcal{B})$ and θ on $\mathcal{B}^a(F_\mathcal{C})$ are stably cocycle conjugate ⇔ they have Morita equivalent PS.*

- C^*–case: Last theorem under countability hypotheses.
- vN-case: Last theorem **and** everything else general.
  (TP $\rightsquigarrow$ vN-TP; total $\rightsquigarrow$ strongly total; cont. $\rightsquigarrow$ strongly cont.)

Spatial E_0–semigroups

Note: ${}_{\mathcal{B}}E^*_{\mathcal{B}^a(E)}$ via $\langle x^*, y^*\rangle := xy^*$ and $bx^*a := (a^*xb^*)^*$.

$$\vartheta_t E \;=\; \overline{\operatorname{span}}\,\mathcal{K}(E)\vartheta_t E \;=\; \mathcal{K}(E)\odot{}_{\vartheta_t}E \;=\; E\odot\underbrace{E^*\odot{}_{\vartheta_t}E}$$

shows $E_t = E^*\odot{}_{\vartheta_t}E =: E^*\odot_t E$.

Theorem

ϑ is spatial $\Leftrightarrow$ $E^\odot$ is spatial.

Proof: "$\Leftarrow$". Put $u_t\colon x\mapsto x\omega_t$. (**Check!**)
"$\Rightarrow$". If u_t fulfills $\vartheta_t(a)u_t = u_t a$, then $u_t\in\mathcal{B}^{a,bil}({}_0E,{}_tE)$. Therefore,

$$\mathrm{id}_{E^*}\odot u_t\colon \mathcal{B} \;=\; E_0 \;=\; E^*\odot_0 E \;\longrightarrow\; E^*\odot_t E \;=\; E_t$$

is a bilinear isometry in $\mathcal{B}^{a,bil}(E_0,E_t) = \mathcal{B}^{a,bil}(\mathcal{B},E_t)$. So, $\mathrm{id}_{E^*}\odot u_t$ is determined by the vector $\omega_t := (\mathrm{id}_{E^*}\odot u_t)\mathbf{1}\in E_t$. $\rightsquigarrow\omega^\odot$. (**Check!**) $\square$

Theorem (MS 2010)

An E_0–semigroup is spatial iff it is spatial as Markov semigroup.

Product systems + unital units $\Longrightarrow$ weak dilations

T Markov iff $\xi^\odot$ unital. (That is, $\langle \xi_t, \xi_t \rangle = T_t(\mathbf{1}) = \mathbf{1}$.)
So, let $E^\odot$ be PS with unital unit $\xi^\odot$.

- $E_t \to \xi_s E_t \subset E_{s+t}$ isometric embedding. (Only right linear!)
- In fact, it is an inductive system. $\rightsquigarrow E = \lim\operatorname{ind}_t E_t$.
- $u_{s,t} \colon E_s \odot E_t \to E_{s+t}$ "survives" to left dilation $v_t \colon E \odot E_t \to E$.
- So, $\vartheta_t(a) \;:=\; v_t(a \odot \mathrm{id}_t)v_t^*$ defines E_0–semigroup on $\mathcal{B}^a(E)$.

Moreover:

- The unit vector $\xi = \xi_t \in E_t \subset E$ fulfills $\xi\xi_t = \xi$.
- So, $b \mapsto \langle \xi, \vartheta_t(\xi b \xi^*)\xi \rangle = \langle \xi_t, b\xi_t \rangle$ defines Markov semigroup.

(**Note:** Of course, E_t can be recovered as $\vartheta_t(\xi\xi^*)E$ with left action $bx_t = \vartheta_t(\xi b \xi^*)x_t$.
Bhat 1996 ($\mathcal{B}(H)$), MS 2002 for arbitrary ϑ.)

We summarize:

Product systems + unital units $\Longrightarrow$ weak dilations

Weak dilations

Theorem (Bhat and MS 2000, MS 2002)

Let $E^\odot$ be a PS with a unital unit $\xi^\odot$. Then the triple (E, ϑ, ξ) is a weak dilation of the Markov semigroup T defined by $T_t := \langle \xi_t, \bullet \xi_t \rangle$:

$$\begin{array}{ccc} \mathcal{B} & \xrightarrow{\quad T_t \quad} & \mathcal{B} \\ {\scriptstyle i=\xi\bullet\xi^*}\big\downarrow & & \big\uparrow{\scriptstyle \mathbb{E}=\langle\xi,\bullet\xi\rangle} \\ \mathcal{B}^a(E) & \xrightarrow[\quad \vartheta_t \quad]{} & \mathcal{B}^a(E) \end{array}$$

Moreover, if $(E^\odot, \xi^\odot)$ is the GNS-construction of the Markov semigroup T, then (E, ϑ, ξ) is the unique minimal dilation of T.

Remark

Works for product systems over arbitrary right-reversible monoids! The crucial ingredient is the GNS-system.

We can say, a weak dilation it is a triple (E, ϑ, ξ) consisting of

- a Hilbert $\mathcal{B}$–module E,
- an E_0–semigroup ϑ on $\mathcal{B}^a(E)$,
- a unit vector $\xi \in E$,

such that $\xi\xi^*$ is increasing $\rightsquigarrow$ $T_t := \langle \xi, \vartheta_t(\xi b \xi^*)\xi \rangle$ is a Markov semigroup of which ϑ, therefore, is a weak dilation.

A weak dilation can be

- strict (strongly continuous) if ϑ is,
- primary if $\vartheta_t(\xi\xi^*) \uparrow \mathrm{id}_E$,
- automorphic, if ϑ is an automorphism semigroup,
- inner if ϑ is implemented as $\vartheta_t = u_t \bullet u_t^*$ by a unitary group.

Note:

- The inductive limit dilation is primary and every primary weak dilation arises that way.
- An inner weak dilation has trivial product system.
- The PS of a weak automorphism dilation is contained in the Picard group of $\mathcal{B}$.

Definition (MS 2006 (preprint 2001) and 2009 (preprint))

A weak dilation $(E, \mathcal{S}, \omega)$ *is a noise if:*

1. E *is a correspondence over* $\mathcal{B}$.
2. $\mathcal{S}$ *leaves* $\mathcal{B} \subset \mathcal{B}^a(E)$ *pointwise invariant.*
3. ω *is central (that is,* $b\omega = \omega b$*).*

Observe:

- A noise is a weak dilation of the trivial semigroup on $\mathcal{B}$.
- A noise also is a unital dilation of the trivial semigroup on $\mathcal{B}$.

Our classification of E_0–semigroups by product systems tells:

Corollary

A strict (normal) E_0*–semigroup on* $\mathcal{B}^a(E)$, E *(strongly) full, is spatial iff is is stably cocycle equivalent to a noise.*

Spatial dynamics and spatial product systems

Spatial product systems $\Longrightarrow$ noises

Recall: $E^{\odot}$ with unital $\xi^{\odot} \rightsquigarrow$

- $E^{\xi} = \lim\text{ind}_t E_t$ over $E_t \to \xi_s E_t \subset E_{s+t}$.
- $E_s \odot E_t \to E_{s+t} \rightsquigarrow v_t \colon E^{\xi} \odot E_t \to E^{\xi}$.
- $E_t \ni \xi_t \mapsto \xi \ni E^{\xi}$ with $\xi\xi_t = \xi$.
- $\rightsquigarrow \vartheta_t^{\xi} := v_t(\bullet \odot \text{id}_t)v_t^*$
  is strict primary weak dilation of $T_t^{\xi} := \langle \xi_t, \bullet \xi_t \rangle$.
- Every strict primary weak dilation arises that way.

Now: $(E^{\odot}, \omega^{\odot})$ spatial product system $\rightsquigarrow$

- Strict primary weak dilation $(E^{\omega}, \mathcal{S} = \vartheta^{\omega}, \omega)$ of $\text{id}_{\mathcal{B}}$
- Since $\omega^{\odot}$ central, left action of E_t survives $\lim\text{ind}_t$
  $\rightsquigarrow$ E^{ω} is correspondence $\rightsquigarrow$ $(E^{\omega}, \mathcal{S} = \vartheta^{\omega}, \omega)$ is noise!
- Every strict primary noise arises that way from its spatial product system $(E^{\odot}, \omega^{\odot})$.

Dilations of Markov semigroups to noises

Theorem (MS 2008 ($\mathcal{B}(H)$), 2009 (preprints))

A (normal) Markov semigroup admits a dilation to a cocycle perturbation of a primary noise iff it is spatial.

Proof.

"$\Longrightarrow$" is obvious. For"$\Longleftarrow$":

- Spatial $T \rightsquigarrow (E^{\odot}, \xi^{\odot}, \omega^{\odot})$
- $\rightsquigarrow (E^{\xi}, \vartheta, \xi)$ weak dilation of T (inductive limit over $\xi^{\odot}$)
- $\rightsquigarrow$ noise $(E^{\omega}, \mathcal{S}, \omega)$ (inductive limit over $\omega^{\odot}$)
- Amplifications of them are cocycle equivalent. $\Longrightarrow$ cocycle!
- Amplifications remain weak dilation and noise, respectively.
- Identification can be done to identify new ξ with new ω. $\square$

Dilations of Markov semigroups to noises

Unclear
- Cocycle adapted?
- Fulfills QSDE?
  - Hudson-Parthasarathy 1984. Uniformly continuous Markov semigroups on $\mathcal{B}(H)$.
    $\mathcal{B}\Big(\Gamma(L^2(\mathbb{R}_+,K))\otimes H\Big)=\mathcal{B}^a\Big(\Gamma(L^2(\mathbb{R}_+,K)\otimes\mathcal{B}(H))\Big)$.
  - Kümmerer-Speicher 1992. Uniformly continuous Markov semigroups on $\mathcal{B}(H)$.
    $\mathcal{B}\Big(\mathcal{F}(L^2(\mathbb{R}_+,K))\otimes H\Big)=\mathcal{B}^a\Big(\mathcal{F}(L^2(\mathbb{R}_+,K)\otimes\mathcal{B}(H))\Big)$.
  - Goswami-Sinha 1999. Uniformly continuous Markov semigroups on $\mathscr{B}\subset\mathcal{B}(H)$.
    Calculus for $\mathcal{B}^a\Big(\Gamma(L^2(\mathbb{R}_+,K)\otimes\mathcal{B}(H))\Big)$ leaves invariant $\mathcal{B}^a\Big(\Gamma(L^2(\mathbb{R}_+,K)\otimes\mathscr{B})\Big)$.
  - MS 2000. Markov semigroups on $\mathscr{B}$ with CE-generator.
    Calculus on $\mathcal{B}^a\Big(\mathcal{F}(L^2(\mathbb{R}_+,F)\Big)$. (No embedding into $\mathcal{H}\otimes\mathscr{B}$.)
  - Köstler 2000. Abstract calculus. (Faithful invariant state!)
  - Many more .... (Sorry!) Also Evans-Hudson flows ....
- Is it possible to write down the cocycle from MS 2000?
- What about the filtration $\mathscr{A}_t=C^*\{u_s\colon 0\le s\le t\}$?

Nonspatial Markov semigroups

- Until recently, in the case $\mathcal{B}(H)$ only type III E_0–semigroup or simple derivations of them.
  Then Floricel 2008, using my construction of an E_0–semigroup for every Arveson system, discovered a Markov semigroup. In MS 2010 (preprint) I showed it is a proper Markov semigroup.
- Fagnola-Liebscher-MS (in preparation): Classical Markov semigroups of Brownian motion and Ornstein-Uhlenbeck are nonspatial. **Even vN!!**
  Cipriani-Fagnola-Lindsay 2000: It admits a (completely!) spatial quantum extension to $\mathcal{B}(H)$.

This raises questions about classification of PS.

- Embedding may change the type.
- Morita equivalence may not chance the type.
- Morita equivalence may change (in the vN-case) the strong type: In fact, the one-dimensional PS of every E_0–semigroup on $\mathcal{B}(H)$ has a strongly continuous unit, but a type III Arveson system has no unit.

Thank you!