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Amenability via actions

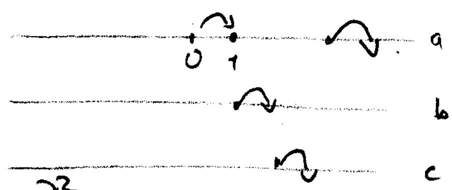
→ New idea: a group acting on a set, orbit, look at it to conclude the group is amenable.
Basic q: can a group admit a faithful action that is amenable while being non-amenable?
Def: Let $W(\mathbb{Z})$ be the wobbling group of $\mathbb{Z}$: bijections g s.t. $|g(j)-j|$ is bounded!

$W(\mathbb{Z}) \curvearrowright \mathbb{Z}$ is amenable because $X_n = [-n,n] \cap \mathbb{Z}$ satisfies

$$\frac{|gX_n \,\Delta\, X_n|}{|X_n|} \to 0 \text{ for every } g \in W(\mathbb{Z})$$ → it is a Følner set/action

$\Gamma = \mathbb{Z}_2 * \mathbb{Z}_2 * \mathbb{Z}_2 \supset \mathbb{F}_2$.
$= \langle a, b, c : a^2 = b^2 = c^2 = e \rangle$

Let w be a reduced word in Γ: ex: $w = abca$
s.t. a, b, c will permute consecutive points! $abca(0) \neq 0$.



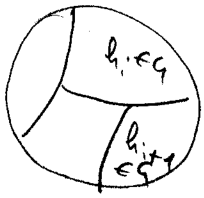
- The action of G is recurrent iff G is virtually 0, $\mathbb{Z}$ or $\mathbb{Z}^2$
- All recurrent actions are amenable.
- If there is a seq. of subsets X_i, increasing, s.t. the border of X_i + neighbours $\subset X_{i+1}$, and such that $\sum |\partial X_i|^{-1} = \infty$, then the action is recurrent.
  Ex: $X_n = [-n, n] \cap \mathbb{Z}$.

pir = product of indep. r.v.

Capacity of an electric network: Woess.

$W(\mathbb{Z}) \ltimes \mathcal{P}_f(\mathbb{Z})$ acts amenably on $\mathcal{P}_f(\mathbb{Z})$ by Woess' theorem.

- Basilica group: is amenable
- Full topologic group of the Cantor minimal system.
- If Γ is finitely generated and simple, then it is not elementarily amenable.



The full topological group of the action $G \curvearrowright \mathcal{X}$ is

gachs

<u>Bratteli diagrams</u> :

level

level

ex. of a rooted tree.

In the theorem: $G \ltimes \bigoplus_X \mathbb{Z}/2\mathbb{Z} \curvearrowright \bigoplus_X \mathbb{Z}/2\mathbb{Z}$ is seldom recurrent.

But amenability should be provable.

Q: is there a natural metric on $W(\gamma)$ that extends the metric of $\mathbb{H}_2$?