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On semigroups of nonexpansive maps
in Hadamard spaces.

Hadamard space = nonlinear metric space.

$A: H \to 2^H$, H Hilbert space, is monotone if $\langle u-v, x-y\rangle \geq 0$ for $x,y \in H$, $u \in Ax$, $v \in Ay$
and maximal if it is w.r.t. inclusion.

Examples. $A = \partial f$, $f: H \to]-\infty, +\infty]$ convex l.s.c.
$A = I - F$, F nonexpansive.

$\dot{u}(t) \in -\partial f(u(t))$

Hadamard space: a geodesic space s.t.

$d(y, x_t)^2 \leq (1-t)\, d(y, x_0)^2 + t\, d(y, x_1)^2 - t(1-t)\, d(x_0, x_1)^2$

the triangles are thinner than in Euclidean space.

for every geodesic $x: (0,1] \to H$, $t \mapsto x_t$

$\to$ negative curvature.

$\Leftrightarrow d(y, \cdot)^2$ is strongly convex.

$|\partial f| = \|Df\|$ in Hilbert space.

$P_C : H \to C$ is nonexpansive.

Notion of weak convergence: is there a corresponding topology?
through projection on geodesics.

"reflexive" space.

Fejér monotonicity: (x_n) is F.m./C if $d(x_{n+1}, c) \leq d(x_n, c)$
for every $c \in C$

Γ-convergence

subdifferential: has a meaning when you are in the case of a manifold.
of curvature -1