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Nonlinear embeddings.

f between metric spaces give rise to ρ_f compression modulus
ω_f expansion — " — of unif. cont.

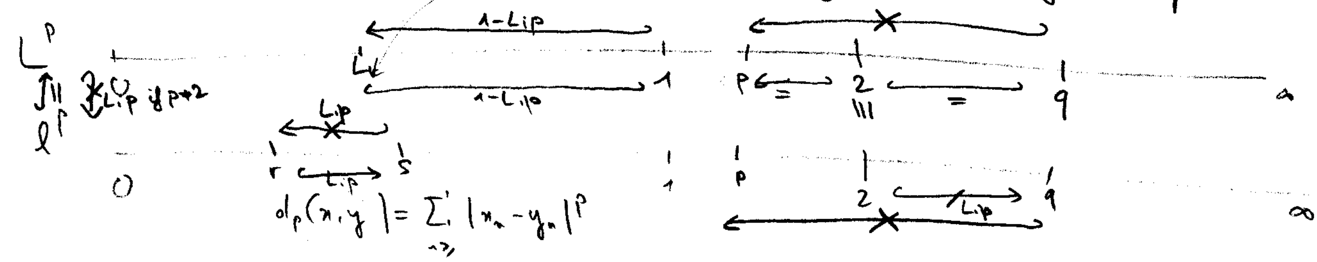
it is a bilip embedding if $\frac{t}{B} \leq \rho_f(t) \leq \omega_f(t) \leq At$.

suppose $|X| = \infty$ and Y Banach space.

quantitative statements have many applications in C. S.

there are notions of coarse and uniform embeddings. Together they give stronge.

no control on the image : ~~image~~ not necessarily a subspace.



Deformation gaps: $X \xrightarrow[\text{coarsely}]{} Y$ with coarse deformation gap $[\rho, \omega]$ if
$\rho \leq \rho_f \leq \omega_f \leq \omega$ whenever t is large enough.

The snowflake exponent is $S_Y(X) = \sup\{0 < s \leq 1 : (X, d^s) \hookrightarrow_{bilip} Y\}$

The compression exponent is $\alpha_Y(X) = \sup\{0 \leq \alpha \leq 1 : X \hookrightarrow_{coarse} Y$ with coarse deformation gap $[t^\alpha, t]\}$

N.B. : ① $0 \leq S_Y(X) \leq \alpha_Y(X) \leq 1$.

② $\underset{\text{equivariant counterpart}}{\alpha^\#_Y(G)} \leq \alpha_Y(G)$ G finitely generated group

We have $\alpha_2(G) \underset{\substack{\uparrow \\ G \text{ amenable}}}{=} \alpha^\#_2(G) \underset{\substack{\uparrow \\ \text{Naor-Peres.}}}{\leq} \alpha^\#_{\ell_p}(G) \leq \alpha_{\ell_p}(G)$

if $G = \mathbb{F}_n$, $n \geq 2$, then $\alpha^\#_2(\mathbb{F}_n) = \frac{1}{2}$ and $\alpha_2(\mathbb{F}_n) = 1$! Guentner-Kaminker cf also Naor-Peres.

Th: • $\alpha_{L_p}(M) = \alpha_{\ell_p}(M)$ if M is locally finite

• $\alpha_2(M) \leq \alpha_Y(M)$ if Y is an ∞-dim Banach space.

Sketch : If M is a locally finite subset of L_p, then
$M \hookrightarrow_{Lip} Y \supseteq \ell_p^n$; use Dvoretzky's theorem.

What about the case M is a group and we consider equivariant embeddings.

$\alpha^\#_2(G) \leq \alpha^\#_Y(G)$

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1) $\ell_2 \underset{\text{strong}}{\hookrightarrow} \ell_p$ + estimates for $p \in]0,2[$.

in particular, $\alpha_{\ell_p}(\ell_2) = 1$ for $1 < p < 2$.

(but 1 is not attained) [Kalton, Randrianarivony].

2) $\ell_s \underset{\text{strong}}{\hookrightarrow} \ell_r$ if $0 < r < s \leq 1$, with good estimates

$$\frac{p}{q} \leq S_{\ell_q}(\ell_p) \leq \alpha_{\ell_q}(\ell_p) \leq \frac{p}{q}$$

Assouad-type embedding. or random + Naor

Kalton - Randrianarivony.

3) $\alpha_{\ell_q}(L_q) \leq \frac{2}{q}$, $q > 2$