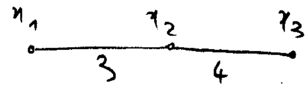
Artin group :

$$\langle x_1,\dots,x_n \mid \overbrace{x_i x_j x_i \cdots}^{m_{ij}} = \overbrace{x_j x_i x_j \cdots}^{m_{ij}} \quad 1\le i\ne j\le n \quad m_{ij}\in\{2,3,\dots,\infty\}\rangle.$$

ex :



$x_1x_2x_1 = x_2x_1x_2$ $\quad m_{12}=3$

$x_2x_3x_2x_3 = x_3x_2x_3x_2$ $\quad m_{23}=4$.

with $m_{ii}=1$, we have the Coxeter matrix $\begin{pmatrix} 1 & 3 & \infty \\ 3 & 1 & 4 \\ \infty & 4 & 1 \end{pmatrix}$

Here a few classes:

1) spherical or finite type if W is finite.

$G = \langle x_1,x_2 \mid x_1x_2x_1 = x_2x_1x_2\rangle$. is the braid group $\twoheadrightarrow \langle x_1,x_2 \mid x_1x_2x_1=x_2x_1x_2,\ x_1^2=x_2^2=1\rangle = S_3$ then it is dihedral finite type.

4) right-angled Artin groups : $m_{ij}\in\{2,\infty\}$.

if $m_{ij}=2$ for all i,j: free abelian group

$=\infty$ —— free group.

The right angled are a mixed.

5) dihedral type if 2 generators.

$G = \langle x$

Q What are the groups with polynomial geodesic factorisation? with the centroid property?