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nonmeasurable approach to QG duality

2 years ago: bounded, or Kac version of this.

G lcq abelian, $\hat{G}$ dual group: $\hat{\hat{G}} = G$.

If G is not abelian, what is the dual? .... QG:

Kustermans and Vaes (2000) defined a w^*LCQG as a vNa M with comultiplication $\Delta: M \to M \bar{\otimes} M$ that is normal and unital. and with left- and right-invariant weights on M: $M_+ \to [0, \infty]$.

then $\hat{M}$ is defined as the vN algebra generated by the left-regular representation of M, i.e. the GNS rep. corresponding to φ.

The $\hat{\hat{M}} \cong M$.

Recall that $C^*(G)$ is defined as the enveloping C^*-algebra of $L^1(G)$.

Q: How to construct the dual without knowing the Haar measure? / weight

Def: $S: M \to M$ is an antipode if

(1) S is densely defined

(2) $(*S)^2 = 1$

(3) S is an antihomomorphism

(4) If $\mu, \nu \in M_*$ and $\mu \circ S, \nu \circ S \in M_*$, then $\Delta(x)(\mu \circ S \otimes \nu \circ S) = \Delta S(x)(\nu \otimes \mu)$

Knowing S, φ one has S, ψ, and thence φ, ψ, and vice versa.

Def: A „QG" is a vnA M with comultiplication and antipode as above

Th: There is a functor on the category of „QG" s.t. $M \mapsto \hat{M}$

N.B.! Q.C. hypergroups of Vainerman + another man from Kiev.

The dual here is reduced and not full.

(1) One has not in general $\hat{\hat{M}} = M$, but $\hat{\hat{\hat{M}}} = \hat{M}$.

(2) one has $\widehat{L^\infty(G)} = W^*(G) = C^*(G)^{**}$ and $\widehat{W^*(G)} = W^*(C_0(G)) \cong C_0(G)^{**}$

(3) If $M = \hat{N}$ and M is commutative, then $M = C_0(G)^{**}$ for some LCG G

If N is commutative, then $N = C^*(G)^{**}$ for some LCG G

(4) If $M = \hat{N}$, then the multiplicative unitary corresponding to M is weakly well behaved.

In the construction, we get $\underset{\text{Banach algebra}}{\phi : M_*} \longrightarrow \hat{M}$ c.b. $*$-homomorphism

and there is a $U \in M \bar{\otimes} \hat{M} = (M_* \hat{\otimes} \hat{M}_*)^*$ s.t. $U(\mu, \omega) = \phi(\mu)(\omega)$

[Mayer-Roi - Woronowicz ?] One shows that U is $\overset{\pi}{M_*} \overset{\pi}{\hat{M}_*}$ unitary.

Conversely, if U is unitary, the cb $*$ map ϕ is unitary.

Prop: $M(G) = M_* = C_0(G)^{**}$

an irrep π is unitary iff $\begin{matrix} t \longmapsto \pi(\delta_t) \\ G \longrightarrow B(H) \end{matrix}$ is continuous.

Let $M_*^0 = \bigcap \{\ker \pi : \pi \text{ irred non unitary}\}$.

We set $\hat{M} = W^*(M_*^0) = C^*(M_*^0)^{**}$

If M_* is the measure algebra $M(G)$, then $M_*^0 = L^{1/2}(G) \underset{\neq}{\supset} L^1(G)$ unless G is discrete

(Joseph Taylor TAMS '69)

but $C^*(M_*^0) = C^*(L^1(G))$: they have the same $*$-rep^s.

free product of two copies of M ... bidual ... what do you get?