Piotr Hajac.

Free actions of compact q groups on unital C^*-algebras.

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Exposant de Hölder et pincement de la courbure. 25/11/2014

metric geometry (1900's). How to get there from Riemann geometry.

First problems in dim 2

Today: ______ 3.

Tomorrow? ∞-dim?

Stop playing with R-metrics and use subriemannian metrics.

ex: Heisenberg group and look at paths that are horizontal from a pt p to a pt q.

Q (Gromov 1993): ~~dilations are~~ identity between Euclidean and Heisenberg is Hölder: one has

$$d_E \leq d_{SR} \leq \sqrt{d_E}$$

But for which exponents $\alpha \in]0,1[$ is there a local homeom between euclidean space and M: let $\alpha(M) =$ sup of such.

Let X metric sp. If top dim $X = n$ and Hausdorff dim $X = Q$, then $\alpha(X) \leq \frac{n}{Q}$ (here $\frac{3}{4}$)

Let M subRiemann ": then $\alpha(X) \leq \frac{n-1}{Q-1}$ (here $\frac{2}{3}$).

Let M further be a contact manifold (odd-dim, hyperplane field max^{ly} nonintegrable) top dim $2m+1$: $\alpha(X) \leq \frac{m+1}{m+2}$

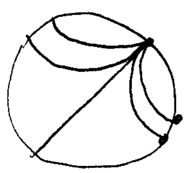
A more complicated problem:

M Riemann $-1 \leq \delta < 0$ M δ-pinched if sectional curvature between -1 and δ ← geodesics have a tendency to diverge exponentially

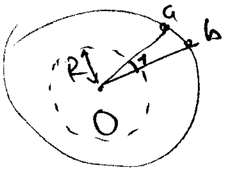
Optimal pinching $\delta(M)$ of $M = \inf\{\delta \geq -1 : M$ q-i to a complete R-manifold, simply connected, δ-pinched$\}$.

Ex: $H^2_{\mathbb{C}}$ hyperbolic plane is $-\frac{1}{4}$-pinched. Is $\delta(H^2_{\mathbb{C}}) = -\frac{1}{4}$?

ideal boundary =



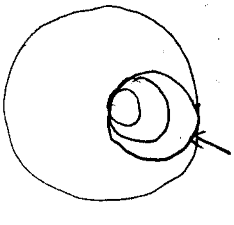
two geodesics are $\sim$ boundary pt if they remain at bounded distance



choose as metric on rays starting from O the radius R such that two radii hit the circle R ~~with~~ in points at distance 1. $d_O(a,b) = e^{-R}$.

Mostow-Margulis 1960's.

The Heisenberg group exists in $PU(n,1)$



horocycle, horosphère. They foliate the space

curvature between -1 and $-\frac{1}{4}$.

optimal pinching should provably be given by the natural metric

quasi-symmetric Hölder-Lipschitz problem: a new exponent a_{qs}

$a_{qs}(\mathrm{Heis}) \leq \frac{2}{3}$. You don't play with distance but with balls
→ like tennis

→ Hyperbolic metric space

Pb with Hölder maps:

$\mathcal{H}^4(\text{open}) \leq C\, \mathcal{H}^3(\partial\, \text{open})^{4/3}$ isoperimetric inequality (Pansu, Varopoulos)

In Heisenberg groups, how to exclude horizontal curves.

NB.: $\int |Du|^n$ (energy) is a conformal invariant, and even a q-conformal one.

And q-s maps are q-c.
↳ and they are almost everywhere differentiable.

Coarea formula: u Lip (= in Eucl. setting): $\int f = \int_{\mathbb{R}} dt \int_{u=t} f/|Du|$

$0 < \int |Du|^n \leq \int_{\mathbb{R}} dt \underbrace{\int_{u=t} |Du|^{n-1}}_{\text{a one-parameter family has dimension at least } n-1}$

but this not known in SR geometry.
Magnani 2002 goes into the direction ≥

One has to replace Hausdorff measures (infimum) by packing measures. mesure d'emplacement (supremum).

then ≥

(N, ℓ)-packing: N multiplicity, ℓ enlargement.

cf. Tricot.

Define the energy of $f^{\omega s}$: $e_u(B) = \text{measure}(u(B))$.

Then there is a coarea $\leq$:

$$E_u^Q(X) \leq \int_M E_u^{Q-1}(u^{-1}(m))\, d\mu(m).$$

($\mathcal{B}$-ling proof).

$\mathrm{meas}(u(B)) \leq \mathrm{diam}(u(B))^d \leq \mathrm{diam}(B)^d$.

→ energy and dimension

Th $\alpha \leq \frac{n-d}{Q-d}$ if $f: \mathbb{R}^n \xrightarrow{C^\alpha} X$
$v: \mathbb{R}^n \to \mathbb{R}^d$ submersion and $u = v \circ f^{-1}$

$E_u^{Q/d} > 0$

→ $E_u^Q > 0$ if packing dim Q.

Are there submersions in $\mathbb{R}^2$?! with $E_u^{Q/2} > 0$. ??

→ hard analysis.

→ then one would have $\leq \frac{n-2}{Q-2}$ ...

Conformally Hölder maps... but turned out to be useless.

Lipschitz assumption very useful

Hölder assumption: very crude.